Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 1 (Group)

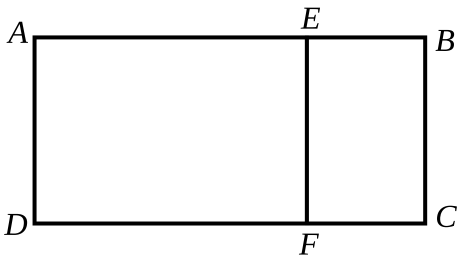
香港数学竞赛 (2006 – 2007)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一， $AEFD$ 是边长为一单位的正方形。长方形 $ABCD$ 的长阔的比例与长方形 $BCFE$ 的长阔比例相同。若 AB 的长度是 W 单位，求 W 的值。

In Figure 1 , $AEFD$ is a unit square . The ratio of the length of the rectangle $ABCD$ to its width is equal to the ratio of the length of the rectangle $BCFE$ to its width . If the length of AB is W units, find the value of W .



图一
Figure 1

2. 在坐标平面上满足 $x^2 + y^2 < 10$，其中 x 及 y 为整数的点 (x, y) 共有 T 个，求 T 的值。

On the coordinate plane, there are T points (x, y) , where x and y are integers , satisfying $x^2 + y^2 < 10$, find the value of T .

3. 设 P 及 $P+2$ 均为质数并满足 $P(P+2) \le 2007$ 。若 S 是符合上述要求的质数 P 的总和，求 S 的值。

Let P and $P+2$ be both prime numbers satisfying $P(P+2) \le 2007$. If S represents the sum of all such possible values of P , find the value of S .

4. 已知 $\log_{10}\left(2007^{2006} \times 2006^{2007}\right) = a \times 10^{k}$，其中 $1 \le a < 10$ 及 k 是整数，求 k 的值。

It is known that $\log_{10}\left(2007^{2006} \times 2006^{2007}\right) = a \times 10^{k}$, where $1 \le a < 10$ and k is an integer. Find the value of k.

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 2 (Group)

香港数学竞赛 **(2006 – 2007)**

决赛项目 **2 (**团体**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $R=1\times 2+2\times 2^2+3\times 2^3+\cdots+10\times 2^{10}$，求 R 的值。

If $R=1\times 2+2\times 2^2+3\times 2^3+\cdots+10\times 2^{10}$, find the value of R .

2. 若整数 x 满足 $x\geq 3+\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}}}$ ，求 x 的最小值。

If integer x satisfies $x\geq 3+\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}}}$, find the minimum value of x .

3. 设 $y=\dfrac{146410000-12100}{12099}$ ，求 y 的值。

Let $y=\dfrac{146410000-12100}{12099}$, find the value of y .

4. 在坐标平面上，某圆以 $T(3,3)$ 为中心及经过原点 $O(0,0)$。若 A 为该圆上的一点使得 $\angle AOT=45^\circ$ 及 ΔAOT 的面积是 Q 个平方单位，求 Q 的值。

On the coordinate plane, a circle with centre $T(3,3)$ passes through the origin $O(0,0)$. If A is a point on the circle such that $\angle AOT=45^\circ$ and the area of ΔAOT is Q square units , find the value of Q .

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 3 (Group)

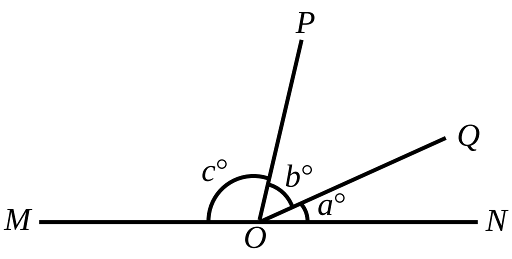
香港数学竞赛 (2006 – 2007)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一，MN 是一直线，$\angle QON = a°$ 、 $\angle POQ = b°$ 及 $\angle POM = c°$。若 $b : a = 2 : 1$ 及 $c : b = 3 : 1$，求 b 的值。

In Figure 1 , MN is a straight line , $\angle QON = a°$, $\angle POQ = b°$ and $\angle POM = c°$. If $b : a = 2 : 1$ and $c : b = 3 : 1$, find the value of b .



图一

Figure 1

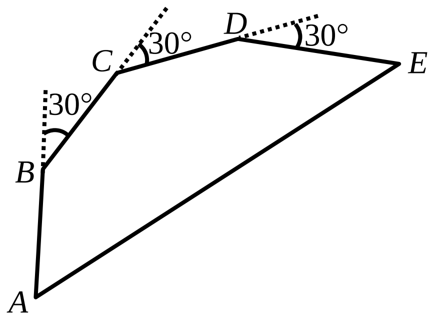
2. 已知 $\sqrt{\frac{50+120+130}{2}\times(150-50)\times(150-120)\times(150-130)}=\frac{50\times130\times k}{2}$ 。

若 $t=\frac{k}{\sqrt{1-k^2}}$，求 t 的值。

It is known that $\sqrt{\frac{50+120+130}{2}\times(150-50)\times(150-120)\times(150-130)}=\frac{50\times130\times k}{2}$. If $t=\frac{k}{\sqrt{1-k^2}}$, find the value of t .

3. 如图二，一蚂蚁由 A 点出发，往前直走 $5\sec 15^\circ$ 厘米至 B 点；接着右转 30°，往前直走 $5\sec 15^\circ$ 厘米至 C 点。蚂蚁再重复右转 30° 及往前走 $5\sec 15^\circ$ 两次，分别到达 D 点及 E 点。若 AE 的距离是 x 厘米，求 x 的值。

In Figure 2 , an ant runs ahead straightly for $5\sec 15^\circ$ cm from point A to point B. It then turns 30° to the right and run $5\sec 15^\circ$ cm to point C. Again it repeatedly turns 30° to the right and run $5\sec 15^\circ$ cm twice to reach the points D and E respectively . If the distance of AE is x cm , find the value of x .



图二

Figure 2

4. 某数学比赛共有 4 条题目。以下述方式为每个题目评分：答对得 2 分、答错扣 1 分、不作答得零分。若至少有 S 名参赛者才可保证比赛中有三人同分，求 S 的值。

There are 4 problems in a mathematics competition . The scores of each problem are allocated in the following ways: 2 marks will be given for a correct answer, 1 mark will be deducted from a wrong answer and 0 mark will be given for a blank answer . To ensure that 3 candidates will have the same scores , there should be at least S candidates in the competition . Find the value of S .

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 4 (Group)

香港数学竞赛 **(2006 – 2007)**

决赛项目 **4 (**团体**)**

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 有糖果 x 粒及 $120 \le x \le 150$ 。将糖果分成小堆，若每堆 5 粒，则余 2 粒；若每堆 6 粒，则余 5 粒。求 x 的值。

   Let x be the number of candies satisfies the inequalities $120 \le x \le 150$. 2 candies will be remained if they are divided into groups of 5 candies each ; 5 candies will be remained if they are divided into groups of 6 candies each . Find the value of x .

2. 在坐标平面上，点 $A(3, 7)$ 及 $B(8, 14)$ 沿直线 $y = kx + c$ 反射，当中 k 和 c 是常数，其像分别是点 $C(5, 5)$ 及 $D(12, 10)$。若 $R = \frac{k}{c}$， 求 R 的值。

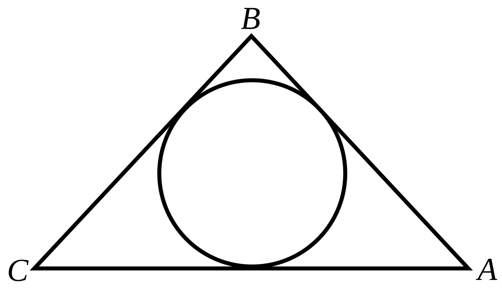
   On the coordinate plane, the points $A(3, 7)$ and $B(8, 14)$ are reflected about the line $y = kx + c$, where k and c are constants , their images are $C(5, 5)$ and $D(12, 10)$ respectively . If $R = \frac{k}{c}$, find the value of R .

3. 已知 $z = \sqrt[3]{456533}$ 是一整数，求 z 的值。

   Given that $z = \sqrt[3]{456533}$ is an integer , find the value of z .

4. 如图一，ΔABC 是一等腰三角形， $AB = BC = 20$ cm 及 $\tan\angle BAC = \frac{4}{3}$。若 ΔABC 的内切圆的半径为 r cm，求 r 的值。

In Figure 1 , ΔABC is an isosceles triangle , $AB = BC = 20$ cm and $\tan\angle BAC = \frac{4}{3}$. If the length of the radius of the inscribed circle of ΔABC is r cm , find the value of r .



图一

Figure 1